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Non-Standard Analysis Without Non-Standard Models

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Leibnitz founded his Calculus on the concept of infinitesimal. It is doubted that he considered infinitesimals as real entities. It was his followers (de l'Hospital and John Bernouille) who emphasized the reality of infinitesimals. Leibnitz himself thought of infinitesimals as ideal or fictious. Or more radically speaking, he thought of them as mere symbols which should be governed by the law of real numbers. (Considering his symbolism, this opinion is highly plausible.) It was Cauchy, through the influence of L. Carnot, who first succeeded to define infinitesimals in an appropriate way , though his definition was still vague for lack of rigorous aspect of variables at that time. His definition of infinitesimal is as follows:

'Lorsque les valeurs numériques successives d'une même variable décroissent indéfiniment, de manière à s'abaisser au'dessous de tout nombre donné, cette variable devient ce qu'on nomme un infiniment petit ou une quantité infiniment petite.'

This definition could bridge a gap between the concept of infinitesimals and "limit", and lead Weierstrass to his rigorous definiton of "limit", that is, so-called ε-δ definition of "limit". Since then, "infinitesimal" had to give its seat to "limit" as a basis of the foundation of Calculus till the emergence of Abraham Robinson.

Early in 1960, Abraham Robinson succeede to introduce infinitesimals in a rigorous and consistent way and develop Calculus by his own method. It

has been shown by himself and his followers that his method, which was called "Non-Standard Analysis" by Robinson himself, is a powerful tool, not only for Analysis but also for many branches of mathematics. An infinitesimal for Robinson is a real entity in a sense that an infinitesimal is an element in a non-standard model, neither a mere symbol nor a variable. Thus, Robinson's aspect for infinitesimals is different from Cauchy's.

Here, let us examine Cauchy's definition of infinitesimals. Usually, one regards a 'variable' in his definition as a dependent variable, and describes it as a function whose limit is 0. However, Cauchy based his theory of limit on the concept of infinitesimals. Therefore, if we admit this opinion it means that Cauchy violated a circular argument. Probably, he had a different picture of infinitesimals in his mind. Could not we imagine that Cauchy thougt of his definition of an infinitesimal as a certain system of a variable x which one can allow to use any conditions "$|x| < \delta$"(δ is a positive constant) whenever the same variable x is concerned?

The aime of this talk is to justify Cauch's definition of infinitesimals along this idea, and to show the possibility that one can dispel non-standard models in Non-Standard Analysis and develop the theory of infinitesimals to the same extent as Robinson developed by using "enlargements".